The Achilles Heel of Plurality Systems: Geography and Representation in Multiparty Democracies

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Building on the unfinished research program of Gudgin and Taylor (1979), we analytically derive the linkage between a party's territorial distribution of support and the basic features of its vote-seat curve. We then demonstrate the usefulness of the corresponding empirical model with an analysis of elections in postwar Great Britain, focusing in particular on the transformation of the Liberals from a territorially concentrated to a dispersed party in the 1970s. We show that majoritarian biases increase with the number of parties, and majoritarian systems harm small parties when their vote is more dispersed than average, and large parties when their vote is more concentrated than average. Moreover, the evolving experiences of Labour and Conservatives demonstrate how a party's territorial support, and hence its expected seat premium or penalty, changes with its electoral fortunes. This model has a wide variety of applications in multiparty majoritarian democracies around the world.

> Three party elections, it seems, can enter the realms of fantasy, making the bias of the cube (or similar) law seem a very tame affair.
>
> —Gudgin and Taylor (1979, 93)

Scholars and pundits are relatively good at predicting the transformation of votes to seats in periods of stable two-party competition in majoritarian democracies. Election-night drama unfolds, however, along with striking asymmetries in the translation of votes to seats, when third parties experience surges in support. Under these conditions, the "winning" party might hope to form an outright legislative majority with less than 40% of the vote. The Canadian Conservatives achieved this in 2011, and the British Conservatives came close in 2010. In countries like India, where multiple parties surge and decline and geographic support bases are constantly in flux, the transformation of votes to seats can be quite difficult to predict.

After observing some sudden and surprising asymmetries in the transformation of votes to seats in Great Britain in the 1970s, Gudgin and Taylor (1979) referred to multiparty competition in their classic book on political geography as the "Achilles heel" of plurality systems of representation. They saw this not only as a normative problem of democratic representation, but also as a puzzle for positivist social science.

It remains both. Much of the literature on majoritarian democracies since then views third parties as annoyances that must be assumed away in order to understand the transformation of votes to seats among the two major parties that stand a chance of forming a government. But this approach misses the mark substantially in the vast majority of majoritarian democracies outside the United States, where winner-take-all districts yield different constellations of party competition in different regions and districts, and where the major parties surge and decline in ways that are not mirror images of one another. Moreover, "third" parties sometimes transform themselves from small, geographically concentrated regional parties to bona fide national parties that compete in all districts. Prime examples are Labor and Social Democratic parties in early 20th-century Europe. Likewise, a party that seems to be fully nationalized after a period of electoral success can, in a period of misfortune, quickly retreat to its original geographic bailiwick.

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Such developments have large and predictable implications for the transformation of votes to seats, and they can sometimes create outcomes that seem stunningly unfair. We know remarkably little about the implications of these patterns of geographic surge and decline for legislative representation in multiparty plurality systems. The literature got off to a good start when Gudgin and Taylor (1979) laid the groundwork for a coherent theory linking the geography of party support to patterns of legislative representation. While shying away from a general model and drawing on historical examples, they made some important observations.

"Gudgin and Taylor showed us the way in the late 1970s," as Johnston (2002, 28) put it, "but very few have followed their signposts." They provided some inductive observations that have stood the test of time, but surprisingly, their intuitions have yet to be transformed into an explicit theoretical and empirical model. Thus, the literature is missing something rather basic: a clear predictive model linking parties' evolving territorial support with the expected number of legislative seats.

This article picks up where Gudgin and Taylor left off and provides such a model. We establish general theoretical and empirical rules that characterize the linkage between a party's territorial distribution of support and the basic features of its seat-vote curve. We analytically derive an *expected majoritarian rate* from the mean and variance of choice probabilities of the multinomial distribution. We define the term $\dot{\rho}$ as the *natural majoritarian bias* and describe its application in empirical models. We provide estimates of its effect and a substantive example of its use.

As the territorial distribution of party votes changes over time, so do the expected majoritarian and partisan biases of electoral rules. We show that majoritarian biases can be approximated by the mean concentration of votes for all parties, whereas partisan bias can be approximated by party-specific deviations from the mean vote concentration.

We present three main analytic findings:

1. Majoritarian biases will increase with the number of parties.
2. Majoritarian systems will benefit small parties when their vote is more concentrated than average and penalize them when their vote is more dispersed than average.
3. Majoritarian systems will penalize larger parties when their votes are more concentrated than average and reward them when their vote is more dispersed.

More importantly, we provide an empirical model that election scholars can implement in a wide variety of settings. Researchers can use our model to assess and predict the transformation of votes to seats associated with a wide range of scenarios as parties enter or leave the electoral arena and as their geographic support base expands and contracts. Our model can help shed light on the transformation of votes to seats associated with historical situations like the rise of workers' parties and the squeezing of Liberal parties in the early 20th century, or the entry and geographic expansion of third parties like the Cooperative Commonwealth Federation (eventually the New Democratic Party) and the Reform Party in Canada. It can also help with pre-election prognostications about seat shares in complex multiparty systems like India's.

Above all, our model also helps shed light on the dynamics of votes and seats in multiparty systems where the basic underlying geographic support base is different for each of the parties, often because of deeply ingrained aspects of political geography, and where the parties can expect very different geographic support distributions as they expand and contract (see Rodden 2013).

We apply the model to British elections from World War II to the present in order to demonstrate its usefulness in two distinct ways. First, we show how the model captures something that has already been intuited by observers of British politics: The Liberal Party has transformed itself from a geographically concentrated minor party to a dispersed minor party, and after the critical juncture of the 1974 election, the Liberals moved from being somewhat advantaged by their relative concentration of votes to being severely penalized after their expansion to new electoral districts.

Second, our analysis of UK elections also illuminates some interesting subtleties about the larger parties that have not been widely discussed in the literature. The dominant approach in the literature has been to ignore the "minor" parties and rely on an unrealistic assumption of uniform swing between Labour and the Conservatives in order to infer the parties' seat shares in hypothetical tied elections. This approach has been increasingly untenable since the resurgence of the Liberals. We show that swings in votes between parties are far from uniform across districts, and the geographic distribution of support for Labour and the Conservatives, and hence their expectations regarding the transformation of votes to seats, are quite different as they grow and contract.

Geography-based partisan biases benefited the Conservatives until recently, since they have been a large party with a vote that was consistently more territorially dispersed than the other parties. Yet we show that Labour's concentrated pockets of support in traditional working-class strongholds allow it to suffer fewer seat losses during its periods in the wilderness (e.g., the Thatcher years) than do the Conservatives during their lean times (e.g., the Blair years). Moreover, Labour's platform moderation in the 1990s allowed it to radically transform its geography, gaining support in moderate Southern districts to achieve heretofore impossible seat premiums.

This example demonstrates how our model can help researchers understand the dynamics of votes and seats as parties expand and contract in other plurality systems. By focusing on hypothetical tied elections, existing research misses out on an important subtlety illuminated by our model: Depending on its geographic support distribution, a plurality system that creates asymmetric seat bonuses for a party during good times can create asymmetric penalties for the same party during bad times (and vice versa).

Geography, Votes, and Seats: An Unfinished Research Agenda

Gudgin and Taylor were part of a larger ongoing effort to understand the properties of vote-seat curves and explain the origins of observed disjunctures between votes and seats. An important goal in this literature is to distinguish between majoritarian bias, whereby the largest party always receives a seat premium, and partisan bias, whereby some parties are expected to receive a larger seat share than other parties with a similar share of the vote. Partisan biases can result from asymmetries in the size of districts (malapportionment), asymmetries in turnout across constituencies, and asymmetries in the partisan distribution of support across constituencies. This article focuses on the latter, which can come about either because of "natural" geographic patterns of party support or intentional gerrymandering. Johnston, Rossiter, and Pattie (1999) and Borisyuk et al. (2010) have provided evidence that the territorial distribution of support across districts is by far the most extensive source of partisan bias in the United Kingdom.

The early work of Brookes (1959, 1960) set the stage for a British and Commonwealth literature that attempts to measure and decompose electoral bias (Johnston 1979, 2002; Johnston, Rossiter, and Pattie 1999). Their approach was to apply a "uniform swing" to district-level election results to examine hypothetical scenarios such as equal or reversed overall national vote shares for the two major parties. If one party receives more seats than the other in equivalent scenarios, the seat differential is classified as partisan bias, which with some algebra can be decomposed into components that are caused by cross-district asymmetries in district size, turnout, and party support.

This approach has yielded important insights. For example, low turnout in Labour strongholds and overrepresentation of Scotland generate consistent bias in favor of Labour, whereas the asymmetries in the geography of support favored the Conservatives (relative to Labour) in the initial postwar period because Labour's support was more geographically concentrated (in urban and mining constituencies) than that of the Conservatives (Johnston 2002).

The entire framework was based on the assumption of two parties, which was no longer tenable after the resurgence of the Liberals in the mid-1970s. Borisyuk et al. (2010) thus extend the original Brookes approach by considering a different type of notional comparison election built on hypothetical scenarios in which each of the three major parties swaps its national vote shares, with the vote swings applied uniformly across all districts. Borisyuk et al. (2010) show that this approach yields substantially different estimates of electoral bias than the two-party approach: In particular, the new estimates suggest that Labour received a substantial seat bonus in the 1980s that was not captured in the simple two-party analysis.

A nagging weakness in this literature is that it does not provide a model of the overall vote-seat curve. Rather, a great deal of work is done by the blunt and often unrealistic assumption that votes swing uniformly across districts from one election to the next. By design, this approach cannot capture the effects of differences in the territorial distribution of support—and hence partisan bias—that are experienced by the same party when it is surging versus when it is declining. As we show below, these differences can be substantial. Our approach is to explicitly model the relationship between territorial support and partisan bias, which affords us (1) an alternative way of measuring party-specific biases, owing to electoral geography that does not rely on highly unrealistic assumptions, and (2) a framework for explaining these biases.

Our approach is also a departure from the American political science literature, which models the relationship between votes and seats but largely ignores electoral geography and multiparty systems. Much of this literature builds from Kendall and Stuart's (1950) statement of the cube law and searches for more general ways of characterizing the relationship between votes and seats (e.g., Grofman 1983; King 1990; King and Browning 1987; Taagepera 1973; Tufte 1973). This literature has given surprisingly little attention to the geography of party support. Until recently, the empirical literature on electoral bias has relied to a surprising extent on aggregate national data rather than district-level data (e.g., Ansolabehere and Snyder 2008; King 1990; Tufte 1973).

More recently, in the spirit of Brookes, Gelman and King (1990) have used transformations of district-level votes to estimate bias and responsiveness, but with the exception of Grofman, Koetzle, and Brunell (1997), this literature has not attempted to disentangle the various sources of partisan bias. Moreover, while U.S. scholars point out that observed asymmetries in the transformation of votes to seats are explained by the more efficient geographic distribution of Republicans across districts (e.g., Chen and Rodden 2013; Erikson 2002), they do not explicitly model electoral geography, and for obvious reasons, U.S. empirical studies have little to say about minor parties. In fact, because of the influence of "Duverger's Law," efforts to extend classic vote-seat models to multiparty systems typically focus on the impact of increasing the district magnitude and moving to proportional representation (e.g., Taagepera 1986; Taagepera and Shugart 1993).

Another closely related literature examines the geographic concentration of support for parties across districts as one of the factors driving the vote share a small insurgent party needs in order to win its first seat (Taagepera 1989, 2002).

We build most directly on the recent work of Calvo (2009) and Linzer (2012). In attempting to model the distortions introduced by the entry of new parties in early 20th-century Europe during the era of franchise expansion, Calvo introduced a generalized version of King and Browning (1987) to examine the impact of party entry in the context of single-member districts with a stable two-party system. However, Calvo (2009) did not model the central concept of this article: cross-party and time-series variation in the geographic concentration of votes. Then, more recently, Linzer (2012) proposed a model of seats and votes that considered the empirical distribution of vote shares at the district level, effectively incorporating district-level support in a model of seats and votes. However, Linzer (2012) does not provide a general theory that explains the allocation of seats as the territorial distribution of support changes. Instead, he takes the territorial distribution of votes as given and conditions the allocation of seats to the different empirical examples observed in the data. Our objective is to provide a more general rationale to explain the expected allocation of seats as the territorial distribution of party votes changes.

In the next section, we introduce readers to a model that explains *majoritarian* and *partisan* biases as a function of the geographic distribution of votes. We distinguish *majoritarian biases* in the overall allocation of seats, which result from differences in the mean party concentration of district-level votes, from *partisan biases* rewarding or penalizing individual parties, which result from cross-party deviations from that mean. This model allows us to describe the evolution of majoritarian and partisan biases over time as parties' support distributions evolve.

The Geography of Seats and Votes in Multiparty Systems

Let us begin with an example. Imagine an electoral system with two parties, a Conservative and a Liberal party, competing in 100 single-member districts that are perfectly apportioned. Every registered citizen casts a vote, with the Conservatives collecting 55% and the Liberals 45%. Let us imagine two different scenarios for this national election:

1. In the first scenario, parties are perfectly dispersed over the space. In any given district, the Conservatives win the same vote share, 55%, while the Liberals collect the remainder, 45%. Given that the Conservatives win a majority of the vote in every district, with just 55% of the vote they collect 100% of the seats. In this scenario, a perfectly dispersed Conservative party wastes no votes and wins every single contest. That is, single-member districts produce a winner-takes-all allocation of seats, and the electoral rules display dramatic majoritarian biases.
2. In the second scenario, parties are perfectly concentrated by district. Therefore, the Conservatives collect 100% of the vote in 55 districts while the Liberals collect 100% of the vote in the remaining 45 districts. Because votes are perfectly concentrated (in perfectly apportioned districts), the Conservatives win 55% of the vote and 55% of the seats, whereas the Liberals receive a seat share that is equal to their vote share. In this scenario, a perfect concentration of votes results in a strictly *proportional allocation of seats with no majoritarian biases.*

The description of these two scenarios illustrates how, in a two-party system, the territorial concentration or dispersion of the vote has a dramatic effect on the majoritarian properties of single-member district FTP rules.

Let us describe the first scenario as a fully nationalized party system, where a party wins roughly the same vote shares in every district, and the second scenario as a fully denationalized or segmented party system, where parties control different regions. As political systems evolve, the territorial distribution of all parties' votes changes and so do the properties of electoral rules.

Similar geographic effects will be shown when there are more than two parties competing for votes. Table 1 previews the expected majoritarian and partisan biases with more than two parties. As we will show, when the distribution of party votes is roughly similar across districts (dispersed vote), majoritarian biases will be larger. When parties concentrate their vote in a few districts, majoritarian biases are attenuated. When the vote of a small party j is more concentrated than the average party, it will win a seat premium (positive partisan biases). By contrast, when the vote of a small party j is more dispersed than the average party, it will be penalized (negative party biases).

Next, we provide a more technical description of our approach, modeling majoritarian biases as a function of the mean dispersion of votes and partisan biases as a function of party-specific deviations from this mean territorial distribution. We begin with a geographic characterization of majoritarian biases in two-party systems and then move on to multiparty systems.

Two-Party Systems

Our basic approach is to modify the inverse logistic equation (King and Browning 1987) that allocates seats to parties as a function of vote shares, allowing majoritarian biases to be expressed as a function of the expected geographic distribution of vote probabilities.

Let us begin by defining two parties, $J \equiv \{L, R\}$, competing for a majority of votes in $k \in K$ single-member districts in an electoral contest $c \in C$. For simplicity, we will assume that each district k selects *Party L* with probability π_L and *Party R* with probability $(1 - \pi_L)$, where vote share $v_L \sim B(1, \pi_L)$ is a random variable with a binomial distribution selecting one party candidate per district, with expected mean $E[v_L] = K\pi_L$, and expected variance $VAR[v_L] = K\pi_L(1 - \pi_L)$.

We will also assume that the winner of each district is elected by a simple majority of votes, with an overall allocation of seats for Party L in contest c derived from the inverse logistic distribution described in King and Browning (1987):

$$S_{cL} = K\left\{1 + exp\left[-\rho\, ln\left(\frac{v_{cL}}{1 - v_{cL}}\right)\right]\right\}^{-1} \quad (1)$$

In Equation (1), S_{cL} describes the total number of seats that Party L expects to collect in the national election c as a function of the probability of winning K single-member districts, the majoritarian parameter ρ, and the overall vote share v_{cL}. Properties of the electoral rules determine the value of the majoritarian parameter ρ, which takes the value of 1 if the system allocates seat shares in proportion to vote shares, $v_{cL} = s_{cL} = S_L/K$. Meanwhile, values larger than 1, $\rho > 1$, provide seat premiums to the winning party, $v_{cL} > \frac{1}{2}$,and seat penalties to the losing party, $v_{cL} < \frac{1}{2}$. Substituting $\rho = 3$ in our two-party system allocates seats as predicted by the venerable cube law, with Party L collecting 23% of seats with 40% of the vote and Party R collecting 77% of the seats with 60% of the votes.

Departing from King and Browning, we will derive a *natural majoritarian parameter* $\dot{\rho}$ in single-member districts solely from the properties of the binomial distribution. We will regard $\dot{\rho}$ as the *Natural Majoritarian Rate* in single-member FTP systems, where the parameter $\dot{\rho}$ is explained by the expected mean and variance of the binomial distribution in K districts. Given that we know the mean and variance of the vote for Party L, we can write the square of the *coefficient of variation*,[1] $CV^2 = \frac{\sigma^2}{\pi_{cL}{}^2} = \frac{v_{cL}(1-v_{cL})}{v_{cL}{}^2} = \frac{1-v_{cL}}{v_{cL}}$ so that $CV = \sqrt{\frac{1-v_{cL}}{v_{cL}}}$, and analytically solve the following:

$$S_L = K\left\{1 + exp\left[-\dot{\rho}\, ln\left(\frac{v_{cL}}{1 - v_{cL}}\right)\right]\right\}^{-1} \quad (2)$$

$$\log(\dot{\rho}) = \alpha\left(\frac{1}{2}\sqrt{\frac{1 - v_{cL}}{v_{cL}}} + \frac{1}{2}\sqrt{\frac{v_{cL}}{1 - v_{cL}}}\right) \quad (3)$$

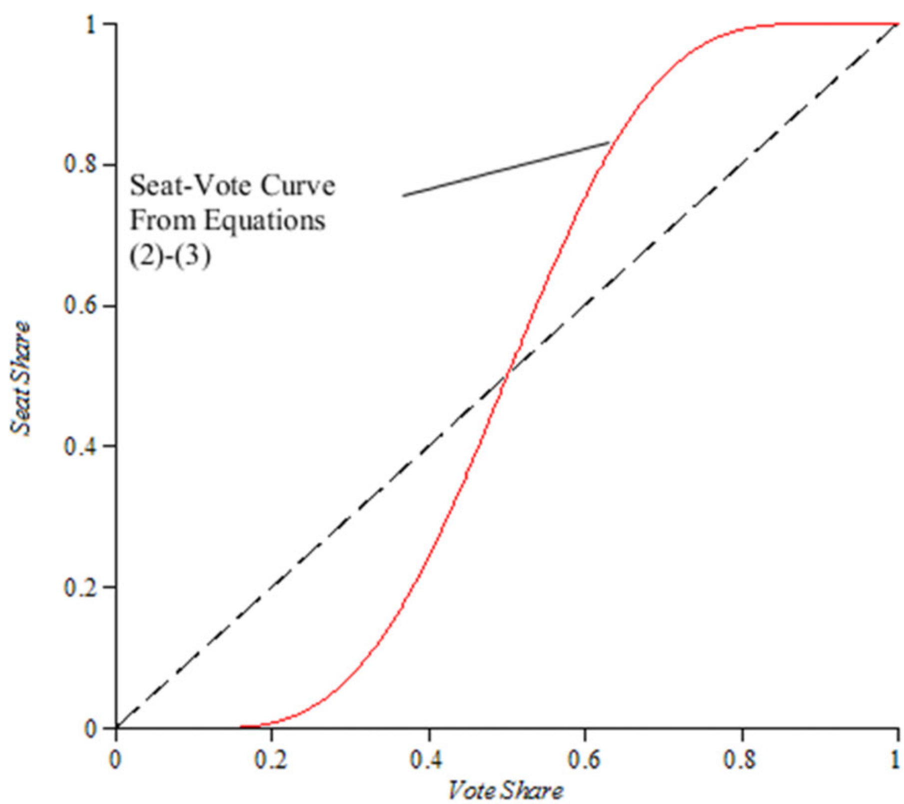
Equation (3) rewrites the change in the majoritarian parameter $\dot{\rho}$ as a function of the mean coefficient of variation in district-level vote shares, using as input the expected mean and expected variance of the binomial distribution for Party L and for Party R. We also add a parameter α that will allow us to rescale the natural majoritarian parameter $\dot{\rho}$ for overdispersed or underdispersed distributions of votes. For the moment, we will constrain $\alpha = 1$ to derive a majoritarian bias that is purely a function of the expected mean and variance in the territorial distribution of vote shares, $\dot{\rho}$.

[1] In probability theory, the coefficient of variation (CV) is a measure of dispersion of a probability distribution. While there are many different measures of dispersion that can be used to assess the geographic distribution of votes, the coefficient of variation provides a simple strategy to incorporate the expected distribution of party votes in the estimation of majoritarian biases. We discuss some advantages to using the Gini coefficient in the empirical section below.

TABLE 1 The General Intuition: Majoritarian Biases, Partisan Biases, and the Territorial Concentration or Dispersion of Party Votes

Party Specific Deviations from the Mean Concentration of Votes	Mean Territorial Distribution of All Parties' Votes: Territorially Concentrated	Mean Territorial Distribution of All Parties' Votes: Territorially Dispersed
$\left\|C_j - \bar{C}\right\| > 0$ Party *j* vote is more concentrated than the average party	•Small majoritarian bias • Positive partisan bias if j is a small party • Negative partisan bias if j is a large party	• Large majoritarian bias • Positive partisan bias if j is a small party • Negative partisan bias if j is a large party
$\left\|C_j - \bar{C}\right\| < 0$ Party *j* vote is less concentrated than the average party	• Small majoritarian bias • Negative partisan bias if j is a small party • Positive partisan bias if *j* is a large party	• Large majoritarian bias • Negative partisan bias if *j* is a small party • Positive partisan bias if *j* is a large party

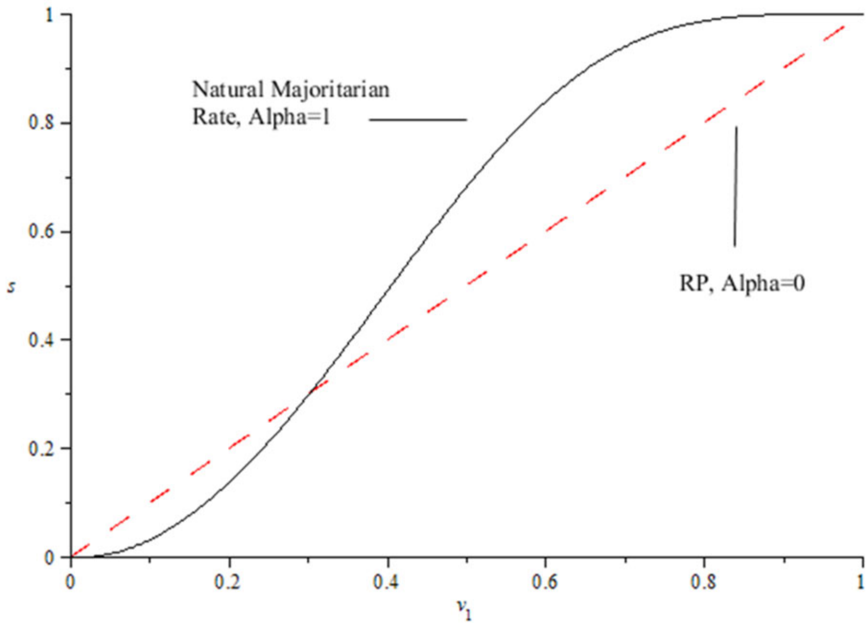
FIGURE 1 Seat-Votes and the Territorial Distribution of the Vote in a Two-Party System



Note: Majoritarian bias is derived from the mean coefficient of variation of the binomial distributions for Party *L* and *R*, $\log(\dot{\rho}) = \alpha\left(\frac{1}{2}\sqrt{\frac{1-v_{cL}}{v_{cL}}} + \frac{1}{2}\sqrt{\frac{v_{cL}}{1-v_{cL}}}\right)$.

Figure 1 plots the expected allocation of seats as a function of vote shares from Equations (2) and (3). The proposed specification makes a number of improvements over previous alternatives. First, it allows for small corrections in majoritarian biases that result from the different level of concentration of district-level votes in different regions of the seat-vote curve. That is, given that we expect the variance in vote shares to be different when two parties approach 50% of the vote or when they are in each extreme (90%–10%), our model induces small corrections that improve on model fit. For example, setting $\alpha = 1$, we may compute the *natural majoritarian bias* when

FIGURE 2 Seat-Votes and the Territorial Distribution of the Vote in a Four-Party System



Note: Majoritarian bias is derived from the mean coefficient of variation of the binomial distributions for Party L and R, $\log(\dot{\rho}) = \alpha(\frac{1}{J}\sum_{j=1}^{J}\sqrt{\frac{1-v_{cj}}{v_{cj}}})$. The solid line describes the natural majoritarian rate, $\dot{\rho}$. Profile for the allocation of votes to all parties is given by $\mathbf{v}_A = \{v_1 = v_1, v_2 = (1 - v_1) * .6, v_2 = (1 - v_1) * .3, v_2 = (1 - v_1) * .1\}$.

Party L collects 30% of the vote (and Party R the remaining 70%) as $\dot{\rho} = 2.97$, very close to the cube law, $\rho_{Jc} = 3$.

On the other hand, when Party L collects 40% of the vote, there is a small decline in majoritarian bias, where $\dot{\rho} = 2.77$. These small differences in seat premiums result from adjustments in the expected variation in vote shares in different areas of the seat-vote curve: that is, from small differences in the probability that a losing party with mean vote $v_{cL} < \frac{1}{2}$ will be able to win more votes than $v_{cL} > \frac{1}{2}$ in at least a few districts.

More importantly, Equations (2) and (3) estimate a theoretical seat-vote majoritarian bias in FTP single-member districts that can be analytically compared to empirical distributions and to natural majoritarian biases derived from other electoral rules. Our model will also allow us to systematically explore majoritarian biases when votes are more or less dispersed than $\pi_L(1 - \pi_L)$, by rescaling $\dot{\rho}$ as a function of α.

It is important to highlight that we can approximate the seat-vote properties of single-member districts without any empirical information about the actual allocation of seats. Indeed, we will show below that our model can approximate the distribution of seats of electoral rules simply by knowing the degree to which the distribution of party votes approximates the mean and variance of the binomial distribution.

Multiparty Systems

We now extend the model to a multiparty setting, allowing for different territorial distributions of party votes, which, as we will show, induce distinct majoritarian and partisan biases. As in the two-party model, majoritarian biases result from differences in the mean territorial concentration of party votes. Different from the two-party model, interparty differences in the territorial concentration of vote shares will induce partisan biases. We will provide structure to these partisan biases, showing that large parties that are concentrated pay a seat penalty while small parties that are concentrated receive a seat premium. This finding is analytically derived, and its comparative statics provide a deeper understanding of the mechanical properties of electoral systems.

Per Equation (3), readers can verify that in the two-party system the value of the majoritarian parameter $\dot{\rho}$ will be identical when a party collects v_{cL} votes or collects its complement $1 - v_{cL}$ (i.e., 40% or 60%). Indeed, with only two parties competing for votes, the expected variances will be symmetric for each of the two parties and the natural majoritarian bias $\dot{\rho}$ will capture all possible allocations of seats. In two-party systems, the distribution of party votes will be symmetric even in the presence of overdispersion or underdispersion, to be rescaled by α. If the vote of one party is territorially dispersed, the vote for the other party will be dispersed as well. Therefore, symmetry in two-party systems will allow the majoritarian parameter $\dot{\rho}$ to capture all biases in the allocation of seats.[2] However, this is no longer the case with more than two parties unless the expected vote probabilities in the multinomial distribution are strictly independent from each other.

Let us begin by generalizing the binomial distribution used in our two-party election to a multinomial design for more than two parties. We now consider $J > 2$ parties with positive vote probabilities, $\mathbf{v} = \pi_{c1} \ldots \pi_{cJ}$, that add up to 1, $\sum_{j=1}^{J} \pi_{cj} = 1$. The expected number of districts that will elect a candidate from Party *L*—out of all possible *K* districts—is $E[v_L] = K\pi_L$, and expected variance $VAR[v_L] = K\pi_L(1 - \pi_L)$. However, the off-diagonal entries in the covariance matrix will reflect the constraint that an increase in the probability of electing a candidate of one party will decrease the probability of electing a candidate from another party, so that $COV[v_1, v_2] = -K\pi_1\pi_2$. This will be important for the statistical implementation of the model but does not affect the more general results that follow.

As in the previous section, we can derive a natural majoritarian bias $\dot{\rho}$ for every combination of vote shares, given that we know the expected mean and variance of each probability in the multinomial distribution. As in the two-party example, we can evaluate the multinomial seat-vote equation:

$$S_1 = K \frac{e^{\dot{\rho}\, ln(v_{c1})}}{\sum_{j=1}^{J} e^{\dot{\rho}\, ln\left(v_{cj}\right)}} \tag{4}$$

$$\log(\dot{\rho}) = \alpha \left(\frac{1}{J} \sum_{j=1}^{J} \sqrt{\frac{1 - v_{cj}}{v_{cj}}} \right) \tag{5}$$

In Equations (3) and (4), the expected seats of Party 1 result from a multinomial distribution with party votes, v_{c1}, the *natural majoritarian parameter* $\dot{\rho}$, and the district magnitude, K. As in the two-party system, the majoritarian parameter $\dot{\rho}$ is a function of the mean coefficient of variation of all parties' votes, $\frac{1}{J} \sum_{j=1}^{J} \sqrt{\frac{1-v_{cj}}{v_{cj}}}$.

For any election, we can map a vote profile, $\mathbf{v} = v_{c1} \ldots v_{cJ}$, to derive the *natural majoritarian bias* $\dot{\rho}$. Let us consider a four-party system as depicted in Figure 2, where we map the vote share of Party 1 while adjusting the other parties' vote at a fixed rate. We begin with a vote profile for election *c* where votes are allocated according to the following rule: $\mathbf{v}_A = \{\mathrm{v}_1 = \mathrm{v}_1,\ \mathrm{v}_2 = (1 - \mathrm{v}_1) * .6,\ \mathrm{v}_2 = (1 - \mathrm{v}_1) * .3,\ \mathrm{v}_2 = (1 - \mathrm{v}_1) * .1\}$.

For example, if Party 1 collects 50% of the votes, then party v_2 collects $\mathrm{v}_2 = (1 - .5) * .6 = .3 = 30\%$ of the votes, party $\mathrm{v}_2 = (1 - .5) * .3 = .15 = 15\%$, and $\mathrm{v}_2 = (1 - .5) * .6 = .1 = 5\%$. This profile allows us to map the expected allocation of seats to votes as Party 1 increases its vote share from 0% to 100%. It will also allow us to show how the majoritarian properties of single-member FTP rules change as a function of the territorial distribution of votes.

Figure 2 describes the natural majoritarian rate as a solid line constraining the parameter to 1, $\alpha = 1$. Consistent with Calvo (2009), as we increase the number of parties from 2 to 4, comparative statics show a sharp increase in majoritarian biases, with the cut point that divides winners and losers moving to the left.[3]

Given the vote profile, we can see that the cut point between winners and losers is approximately 37% of votes, with parties above this threshold winning a premium in seats and parties below this threshold suffering a penalty. Because the variance of the multivariate normal is narrower at the tails, majoritarian biases rapidly increase as the vote becomes more fragmented. Indeed, a rapid increase in the coefficient of variation can be equated with more homogeneity across districts and more dramatic seat premiums.

Plugging some values in Equations (4) and (5) will be illustrative. Consider, for example, that we have four parties with vote shares, $\mathbf{v} \equiv \{\mathrm{A} = .4, \mathrm{B} = .3, \mathrm{C} = .2, \mathrm{D} = .1\}$. Given this profile, we may compute an effective number of electoral parties of 3.33 (e.g., $\frac{1}{.4^2+.3^2.2^2.1^2} = 3.33$) and, substituting proper values in (4) and (5), note that Party A wins 51.3% of the vote with 40% of seats. In the next election, imagine that half of Party B voters defect to Party D, so that vote shares are allocated $\mathbf{v} \equiv \{\mathrm{A} = .4, \mathrm{B} = .2, \mathrm{C} = .2, \mathrm{D} = .2\}$. The fact that Party A has a larger lead has several different implications: First, the effective number of electoral parties increases to 3.57 (e.g., $\frac{1}{.4^2+.2^2.2^2.2^2} = 3.57$). Second, the increase in the effective number of electoral parties increases the majoritarian bias in the system by moving

[2] In other words, our theoretical model intentionally turns a blind eye to partisan bias arising from skewed distributions of party votes in two-party systems. We return to this issue below.

[3] Calvo (2009) shows that the cut point between winners and losers moves exactly to $cutpoint = \sum v_i^2$.

the seat-vote curve to the left. Third, substituting the new values into Equations (4) and (5), we can see that Party A has increased its seat advantage to 53.3%: 2% more seats than in the previous election. Indeed, although Party A's vote total has remained unchanged, increased fragmentation of the vote among the three opposition parties increases the seat premium of Party A.

Let us explain in further detail why an increase in the number of parties yields larger majoritarian biases in single-member FTP systems. Readers will have already noticed that the *natural majoritarian parameter* $\log(\dot{\rho}) = \alpha(\frac{1}{J}\sum_{j=1}^{J}\sqrt{\frac{(1-v_{cj})}{v_{cj}}})$ will increase as the variance $(1 - v_{cj})$ becomes larger relative to the mean party vote, v_{cj}. In a two-party system, the coefficient of variation is large for the losing party and small for the winning party. However, as the number of parties increases, there are more parties whose mean vote share falls below 50%. That is, the mean vote share in the denominator v_{cj} becomes smaller and the variance larger, leading to larger values of CV. This feature of the model captures the essential characteristic of multiparty competition in polities with single-member districts: A party that collects 60% of the vote and faces four parties with 10% of the vote each will do considerably better than a party with 60% of the vote that faces a single opposition party collecting 40% of the vote. Indeed, vote fragmentation increases the rewards for successful parties, as the cut point for victory moves to the left of 0.5, making large margins of victory increasingly valuable to large parties and a burden to small parties.

In real applications, as we will show, votes tend to vary more widely across districts, yielding majoritarian biases that deviate from the natural majoritarian rate of the multinomial distribution. As in the two-party example, we can adjust the natural majoritarian bias by allowing α to take a number of different values that rescale $\dot{\rho}$. When $\alpha = 0$, then $\dot{\rho} = 1$ and the allocation of seats is proportional to votes. When $\alpha = 1$, the model reduces to the expected allocation of seats analytically derived from the mean and variance of the multinomial distribution, $\dot{\rho}$.

Parties often have a distribution of district-level votes that is more concentrated or dispersed than that of their competitors, the result of contextual and behavioral factors affecting the probability rate in different geographic regions. Heterogeneity and homogeneity in district-level votes results in coefficients of variation that are larger or smaller than expected—that is, coefficients of variation that differ from the natural rate, $\widehat{CV} \neq \dot{CV} = \sqrt{\frac{1-v_{cj}}{v_{cj}}}$.

Equations (6) and (7) describe the full model, which incorporates both the geographic determinants of the majoritarian bias and of partisan biases. Because the variance will no longer be constant across districts, we will now consider a mean district vote share $\overline{v_{ckj}}$ for Party j that is different from the national vote share v_{cj}, such that $\overline{v_{ckj}} \neq v_{cj}$. Furthermore, to simplify the notation, let us define a coefficient of variation of party j that may differ from its natural rate, $\dot{CV}\,\widehat{CV}_j = \frac{\dot{CV}}{\delta}$, by some multiplicative factor δ_j, capturing variances that are larger than its natural rate, $\delta_j < 1$, or smaller than its natural rate, $\delta_j > 1$.

To describe how asymmetries in the geographic concentration of the vote affect expected party seats, let us augment our model so that

$$S_{jc} = K\frac{e^{\dot{\rho}\, ln(v_{jc})}}{\sum_{j=1}^{J} e^{\dot{\rho}\, ln(v_{jc})}} \text{and} \tag{6}$$

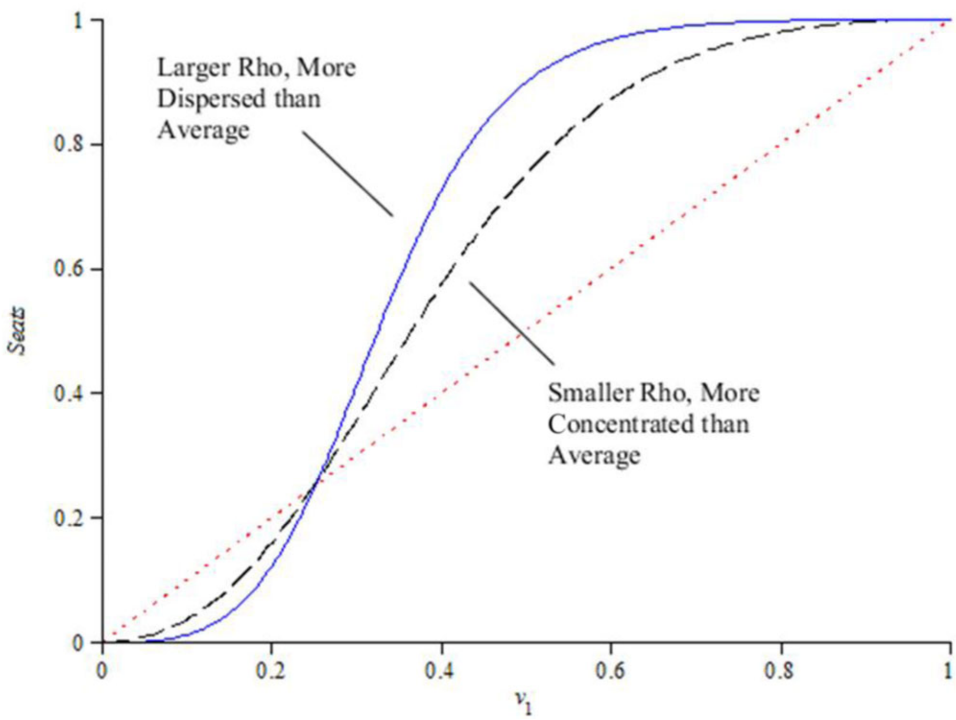
$$\log(\rho_{cj}) = \alpha_1\left(\frac{1}{J*C}\sum_{c=1}^{C}\sum_{j=1}^{J}\widehat{CV_{jc}}\right) - \alpha_2\frac{e^{\widehat{CV_{jc}}}}{\sum_{j=1}^{J} e^{\widehat{CV_{jc}}}}. \tag{7}$$

Our final model includes a change in majoritarian biases that is a function of the mean increase in the territorial concentration of party votes, $\alpha_1(\frac{1}{J*C}\sum_{c=1}^{C}\sum_{j=1}^{J}\widehat{CV_{jc}})$. However, Equation (7) also includes a term measuring the relative concentration of Party j with respect to all other parties, $-\alpha_2\frac{e^{\widehat{CV_{jc}}}}{\sum_{j=1}^{J} e^{\widehat{CV_{jc}}}}$, where $-\alpha_2$ is negative given that a more concentrated party vote, $\widehat{CV_{jc}} > \widehat{CV}_{\neg jc}$, will result in a lower partisan bias for party j in election c, $\rho_{cj} < \rho_{c\neg j}$.

Figure 3 describes two parties with different levels of concentration of party votes: a more concentrated party, $CV_{cj} > CV_{\neg jc}$, and a more dispersed party, $CV_{jc} < CV_{\neg jc}$. The lines intersect at the cut point that divides the winners from the losers of the electoral contest, with parties above the cut point receiving premium seats when dispersed and losses when concentrated. Meanwhile, small parties receive premium seats when concentrated and seat losses when dispersed.

Equations (6) and (7) provide a formal explanation of why winning parties with a concentrated vote are penalized and winning parties with a dispersed vote are rewarded in multiparty elections. In the next section, we estimate the proposed model using data from the United Kingdom since 1950. We show parameter estimates that adjust to expectations and provide a systematic explanation for (1) the larger seat penalties suffered by the Liberals since 1974 and (2) the contrasting experiences of

FIGURE 3 Seat-Votes and the Territorial Distribution of the Vote in a Four-Party System, with Partisan Bias for a Party *j* and a Party ¬ *j* from Differences in the Coefficient of Variation among Parties



Note: Partisan bias is derived from differences in the mean coefficient of variation among parties, $\log(\rho) = -\alpha_2 \frac{e^{\widehat{CV_{jc}}}}{\sum_{j=1}^{J} e^{\widehat{CV_{jc}}}}$.

Labour and Conservatives during their respective periods of dominance and distress.

An Example: The Geographic Distribution of Votes in the UK

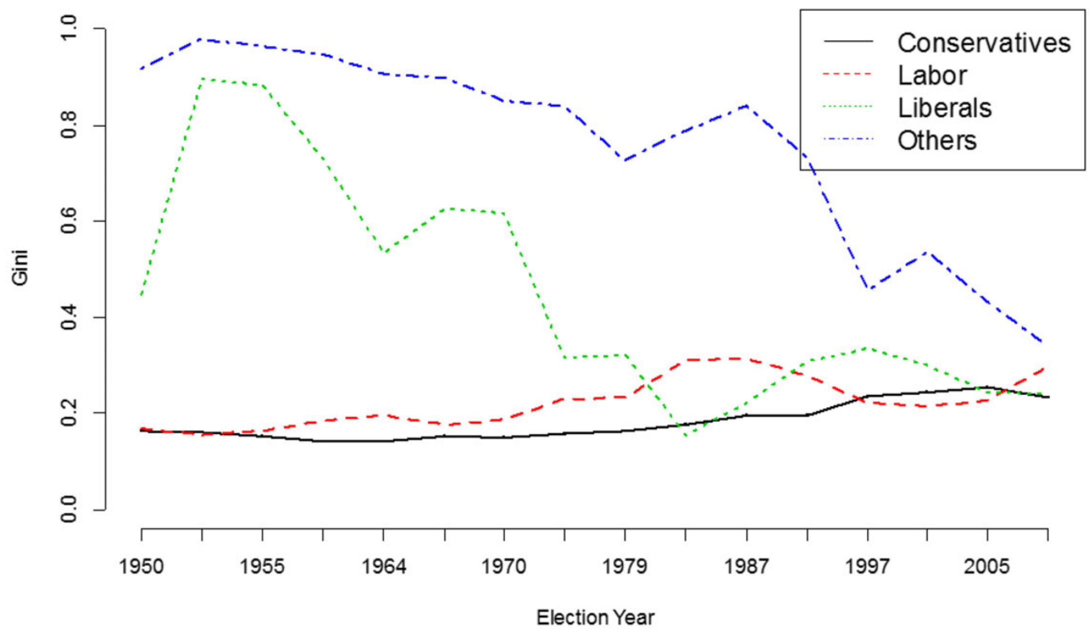
To demonstrate the relationship between the territorial distribution of votes and the allocation of seats, we analyze the evolution of party votes in the United Kingdom since 1950. Figure 4 describes the concentration of party votes with Gini coefficients. Each of the three main parties has followed different trajectories.

Beginning in 1950, the Conservatives have consistently maintained a nationalized party vote, with relatively low but slowly increasing levels of concentration. Labour, on the other hand, started with an identical level of nationalization, but it has steadily become more concentrated than the Conservatives, especially after 1970, with Liberals chipping away its votes in Birmingham and Conservatives chipping away votes in Southern districts. More recently, New Labour has been able to dramatically (but temporarily) reduce its concentration, even achieving a slightly more dispersed support base than its competitors during the Blair years for the first time in history. Figure 4 also reveals that the Gini coefficients of Labour and Liberals are much more variable than that of the Conservatives.

More importantly, we see that since 1974, all three major parties are roughly within the same band, as the Liberals have achieved a far more geographically dispersed support base. The Liberals have undergone a dramatic transformation from a losing but geographically concentrated party to a losing but geographically dispersed party, with important implications for the entire party system.[4]

[4]For more information on the evolution of Gini coefficients, see the Lorenz curves in online Appendix A.

FIGURE 4 Territorial Concentration of the Party Votes in the United Kingdom in the Postwar Years, with Gini Coefficients for Conservatives, Labor, Liberals



Note: Calculated from district-level vote data from Caramani (2000) and *The Guardian*.

Table S1 (in the supporting information) describes the key variables of interest: the overall party vote, the mean district-level party vote, the standard deviation, and the coefficient of variation of the parties. Table S2 describes the difference between the theoretical and empirical coefficient of variation. Armed with these variables and our theoretical model, we are now ready to estimate a statistical model that captures the role of geography in determining whether parties are benefited or penalized in the transformation of votes to seats.

The Statistical Model

The statistical implementation of the proposed model takes as its dependent variable the total number of seats S_{jc} won by party j in election c in each of the 16 elections from 1950 through 2010. As independent variables we include the national vote share v_{jc} and the Gini for each party j in election c. In the theoretical model, it was expedient to use the coefficient of variation to capture vote concentration, but in empirical applications we can also use the Gini, which is more typically used in existing research such as Jones and Mainwaring (2003). Further, Gudgin and Taylor (1979) and Rodden (2013) argue that skewed cross-district distributions emerge naturally from the geography of the industrial revolution since voters for workers' parties tend to be clustered in former industrial zones. These skewed distributions are better captured by the Gini than the CV. Inspired by the same theoretical model, we estimate the empirical model relying on the Gini from Figure 4 rather than the CV as the indicator of territorial vote distribution. However, all analyses have also been replicated using the coefficient of variation and are presented in Figure S1 in the supporting information.

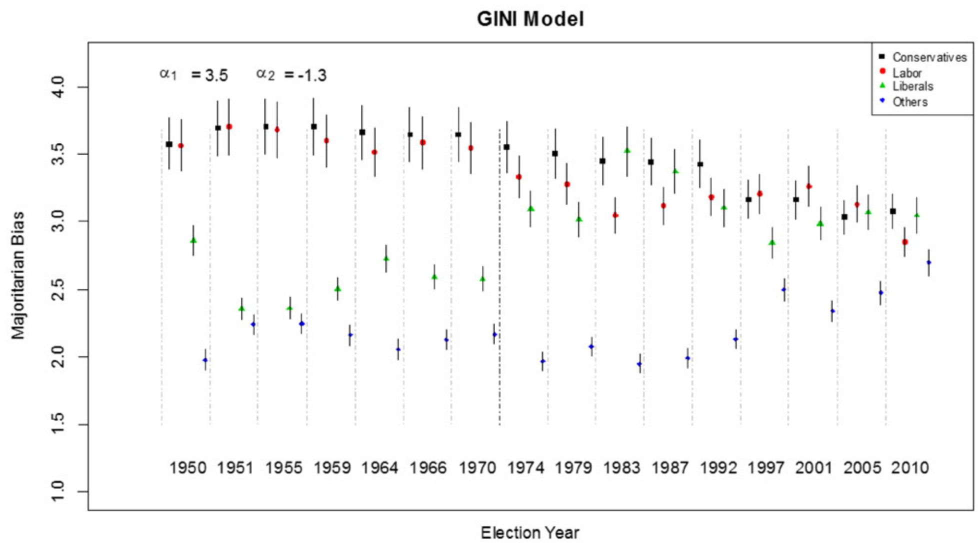
Based on Equations (6) and (7), we write our statistical model in Equations (8) and (9), substituting the Gini coefficient for the coefficient of variation. We implement the model using a Bayesian Markov chain Monte Carlo (MCMC) design in Winbugs 1.4.1:

$$S_{jc} = \frac{e^{\rho_{cj} ln(v_{jc})}}{\sum_{j=1}^{J} e^{\rho_{jc} ln(v_{jc})}} \tag{8}$$

$$\log(\rho_{jc}) = \alpha_1 \left(\frac{1}{J * C} \sum_{c=1}^{C} \sum_{j=1}^{J} G_{jc} \right) - \alpha_2 \frac{G_{jc}}{\sum_{j=1}^{J} G_{jc}} \tag{9}$$

The model estimates the expected allocation of seats, S_{jc}, for each party as a function of the share of party votes, v_{jc}, and the territorial concentration of votes measured by the Gini coefficient, G_{jc}. The model estimates a

FIGURE 5 Majoritarian and Partisan Bias, UK Elections, Selected Parties, 1950–2010



Note: Estimates of ρ_{cj} by party and election. Lower values of ρ_{cj} for individual parties indicate majoritarian attenuation, providing seat premiums to small parties and seat losses to large parties. In 1974, the Liberals completed the transition from a small party with a concentrated vote to a small party with a dispersed vote, losing the seat premiums they enjoyed from 1950 through 1970. Estimates from the CV model, all parties, are in the supporting information, Figure S1. Estimates for all parties, Gini model, are in the supporting information, Figure S2.

majoritarian bias ρ_{cj} by party j and election c. This party-specific majoritarian bias changes as a function of each party's territorial concentration of votes. Consistent with the model, we expect $\alpha_1 > 0$, with majoritarian biases increasing as a function of the mean Gini for all parties, $\bar{G}_c = (\frac{1}{J*C}\sum_{c=1}^{C}\sum_{j=1}^{J} G_{jc})$. Consistent with the model we expect $\alpha_2 \frac{G_{jc}}{\sum_{j=1}^{J} G_{jc}} < 0$, with partisan biases benefiting small parties that are territorially concentrated and penalizing small parties that are territorially dispersed, $\rho_{jc} < \rho_{\neg jc}$. There are no other moving parts in the model, with mean majoritarian bias increasing as a function of α_1 and party-specific biases decreasing as a function of α_2.

Results

Results of the model conform to model predictions, with the median posterior estimate of $\alpha_1 = 3.5$ [3.42, 3.54], with 80/20 intervals in brackets; and the median posterior estimate of $\alpha_2 = -1.34$ [–1.38, –1.29]. The median majoritarian parameter for the entire period is $\rho_{cj} = 3.01$, slightly larger in the 1950s and slightly smaller by 2010. Estimates of model fit in online Appendix C show significant improvement over the restricted model with a fixed majoritarian parameter ρ, from Equation (1), with deviance[5] decreasing from 706 to 450, representing an improvement of 34%.

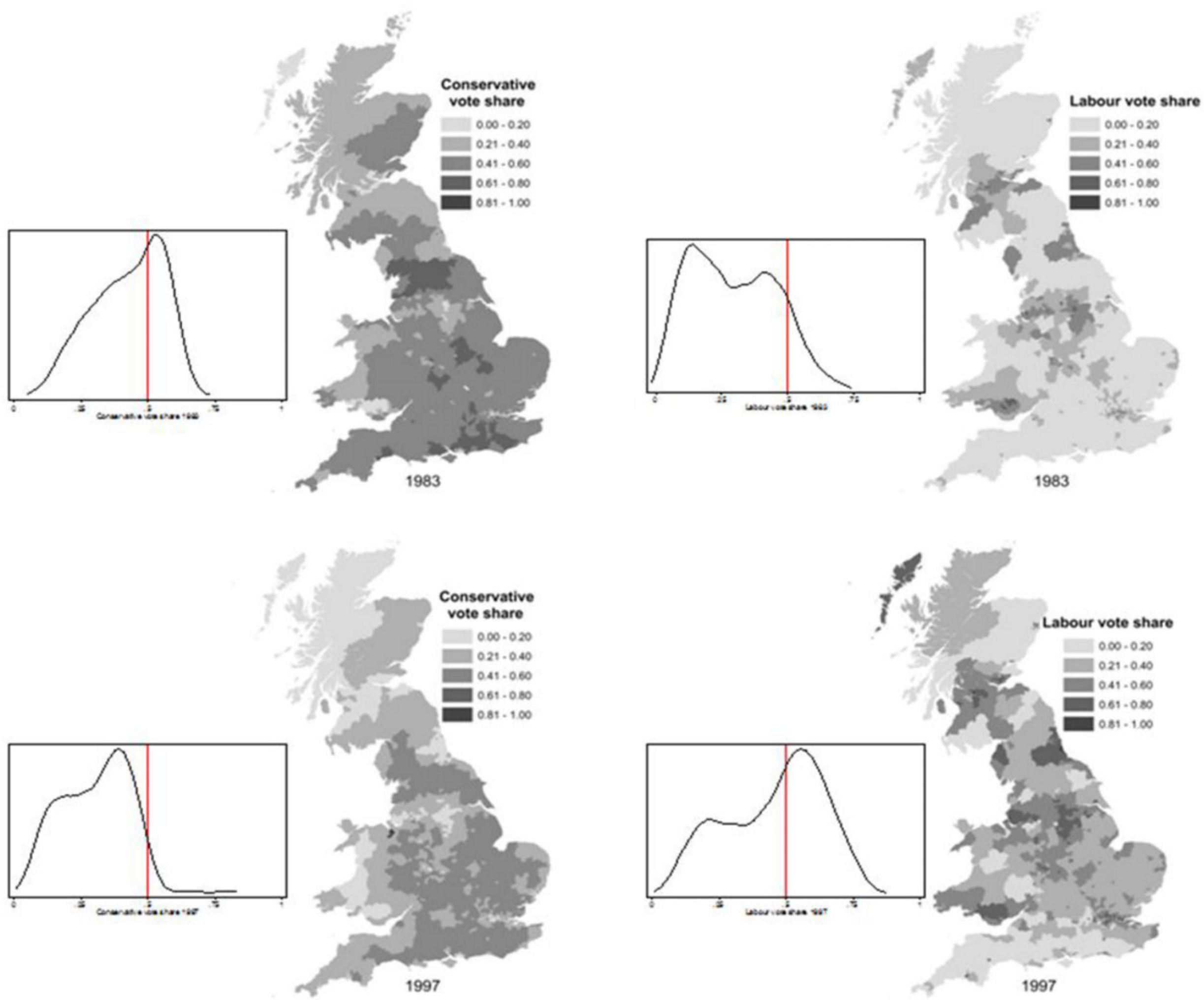
To help visualize changes over time, Figure 5 plots ρ_{jc}, the individual bias parameters for each party and election.[6] In order to interpret Figure 5, it is useful to refer back to the vote-seat curve in Figure 3. A high value of rho translates into a large seat premium for parties above the winning threshold while magnifying seat losses for those below the winning threshold. The Liberals have always been below the threshold, so the increase in rho in the 1970s had the effect of widening the gap between their seats and votes.

The slight overall decline in majoritarian bias is explained by the increased concentration of party vote for Labour and the Conservatives. Figure 5 shows that party-specific biases for Labour and the Conservatives are becoming milder as they become more geographically

[5]Deviance $D(y) = -2(\log(p(y|\widehat{\theta_r})) - \log(p((y|\widehat{\theta_u})))$ provides an assessment of goodness of fit for Bayesian models, similar to a likelihood ratio estimate in maximum likelihood, where $\widehat{\theta_r}$ describes the model parameters of the restricted model and $\widehat{\theta_u}$ describes the estimates of the full model. The full description is in Appendix S3.

[6]Figure 5 is based on the Gini model. In order to verify that the CV model is substantively similar, see Figure S1 in the supporting information.

FIGURE 6 The Geographic Distribution of Labour and Conservative Votes during Good Times and Bad Times



concentrated, offsetting the increases for the Liberals as they become less so.

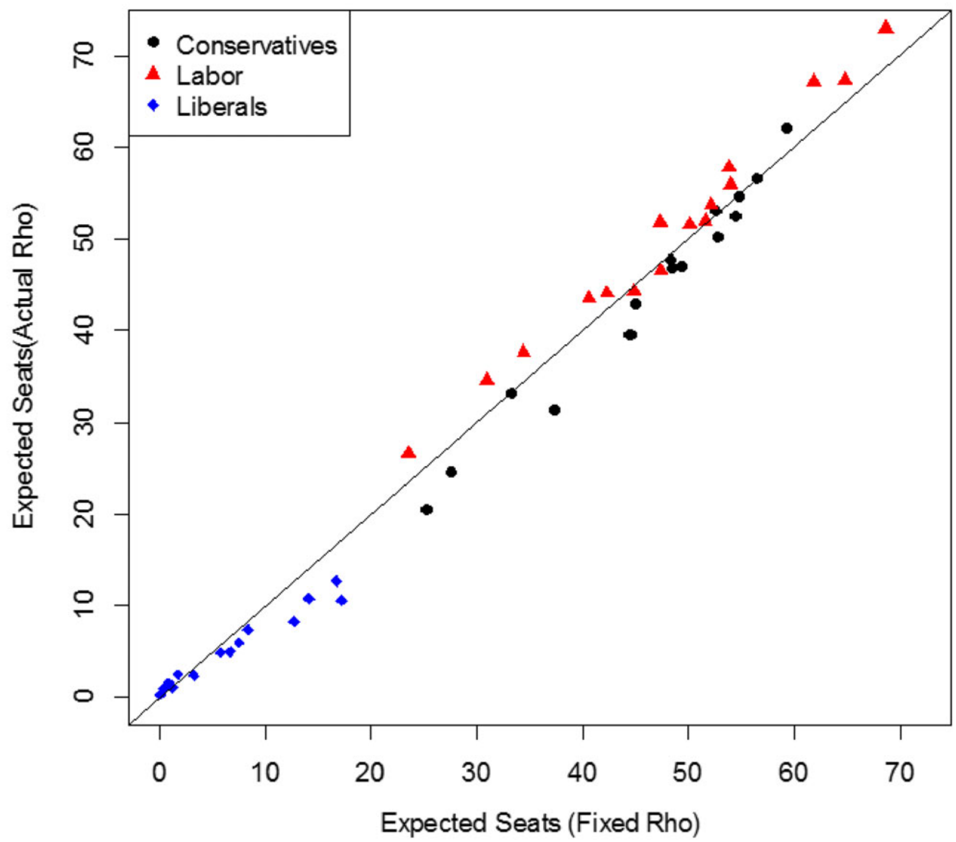
Because of its relatively dispersed support distribution, estimated rho was consistently higher for the Conservatives than for Labour prior to the platform moderation of New Labour. After the geographic spread of the Liberals in the 1970s and the beginning of Labour's lengthy period of electoral misfortune, an interesting pattern emerges in which rho for Labour is far below the estimate for the Conservatives.

Thus, as Labour loses support, its generally more concentrated support base has a silver lining that is captured by our model. Large swings away from Labour, such as during the Thatcher period, yield relatively small seat losses because Labour is able to retreat into its bunkers and hold onto its core working-class seats, effectively becoming a losing but territorially concentrated party. This can be seen in the rather low level of rho for the 1983 landslide and the subsequent significant period in the wilderness for Labour. Even as it lost significant votes to the Conservatives and Liberals, its seat losses were relatively modest.

While Labour is protected during bad times, it can also hope for large seat premiums during good times. We see that "good times" only come for Labour when its support base is just as diversified as that of the Conservatives (Figure 4). Figure 5 shows that estimates of rho for Labour and the Conservatives were rather similar during the Blair era. Thus, the Conservatives were suffering large seat losses and New Labour was receiving seat bonuses.

This suggests a potentially important difference between the geographic support distributions for the major British parties that has not been emphasized in the existing literature. The difference is driven home by Figure 6, which provides maps as well as kernel

FIGURE 7 Expected Seats from Models with Varying versus Fixed Rho



Note: Estimated from model parameters in Equations (8) and (9) and Figure 5. Model comparison is in online Appendix C.

densities representing the cross-district support distributions for Labour and the Conservatives, each at their postwar zenith/nadir in 1983 and 1997.

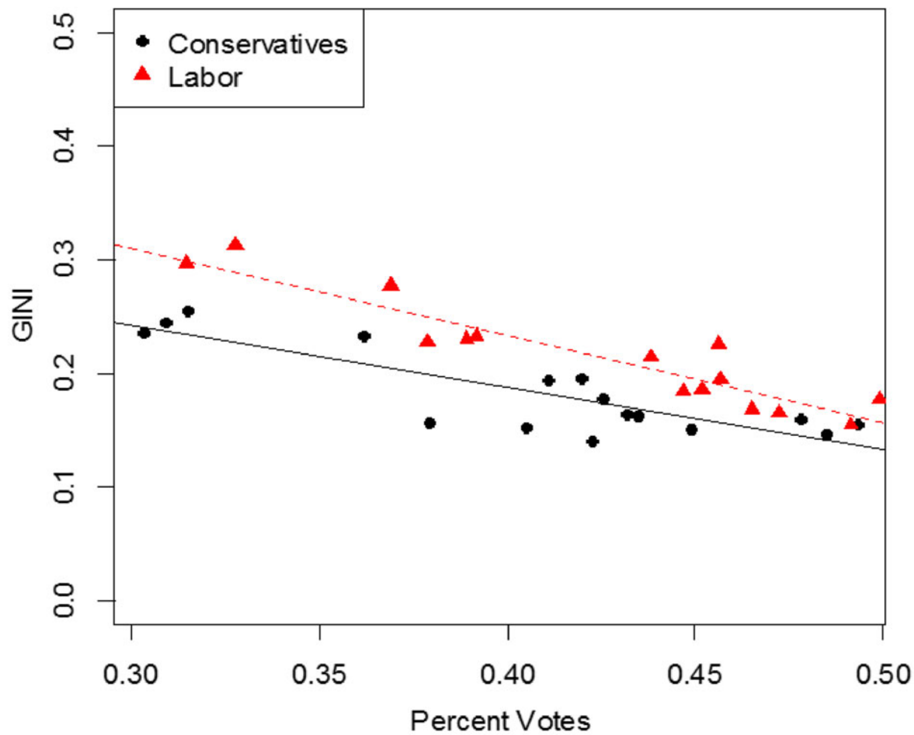
In 1983, Labour reached rock bottom, with 30.8% of the vote, and in 1997, the Conservatives reached their low point with 30.7% of the vote. Yet in spite of these similar vote shares, Labour won 229 seats in 1983 compared with the Tories' 165 in 1997. Figure 6 explains the difference. Even at its worst moment, Labour still had comfortable majorities in a large number of its core urban and mining seats.

The Conservatives do not have a similar line of defense. In good times and bad times alike, Figure 6 shows that they have a relatively similar left-skewed support distribution. Note the peak of the distribution in 1983 just above 50%. When the Conservative brand name suffers, the entire distribution simply shifts to the left, and a large tide can do severe damage to their parliamentary representation.

When Labour made its impressive gains in the 1990s in the suburban Southern districts, its support distribution did not merely shift rightward, as "uniform swing" models would have it. Rather, Figure 6 shows that the shape of the distribution transformed altogether as Labour became substantially more geographically dispersed. Recall the related phenomenon from Figure 4 above: Labour's geographic support distribution is far more variable from one election to another than that of the Conservatives, and the rise of New Labour reversed what had been an upward creep in geographic concentration.

Figure 7 helps relate these observations back to our empirical model. It is difficult to interpret the election-specific estimates of rho in Figure 5 since the overall benefit imparted or pain inflicted on a party by the majoritarian bias parameter is a function of its vote share in the specific election. Thus, Figure 7 provides a plot of expected seats from the model with year- and election-specific rho (on the vertical axis) against the expected seats from a model in which rho is fixed across all elections and parties. Accordingly, observations above (below) the 45-degree line are those where a party's geographic support

FIGURE 8 Vote Shares and the Territorial Distribution of the Conservative and Labor Votes, 1950–2010



Note: Calculated from district-level vote data from Caramani (2000) and *The Guardian*.

distribution makes it better (worse) off than would be expected if it had the mean geographic support distribution.

Figure 7 shows that when Labour is doing badly and can expect to receive less than 40% of the seats, the expected seat share owing to its concentrated support geography is better than what would be expected if it had the mean territorial support distribution (the red triangles are above the 45-degree line). Conversely, when times are tough for the Conservatives, they can expect to do worse than if they had the mean territorial support distribution. In close elections around the cut point separating Labour and Conservatives, asymmetric partisan bias cannot be discerned—the red and black markers are tightly clustered around the 45-degree line. As for the Liberals, Figure 7 shows that as they gained support and diversified, a small seat premium evaporated and became a seat loss.

Finally, the dynamics between the two major parties can be further understood with a scatterplot of the Gini coefficient against vote shares by party, presented in Figure 8. It shows that Labour's support is more concentrated in general, but the gap is largest when contrasting landslide elections. For both parties, the territorial concentration of support diminishes as the party gains votes. However, the slope is steeper for Labour, and as the graph approaches 50%, the difference between parties evaporates. When Labour gains votes, it does so by diversifying its support base so that its distribution is indistinguishable from that of the Conservatives. When Labour loses votes, it retreats into its working-class bunkers. That is, vote losses are more asymmetric for Labour and more evenly spread across districts for the Conservatives.

By demonstrating these important asymmetries across the major parties as the Liberal Democrats have diversified, our analysis casts doubt on the practice of artificially and uniformly shifting votes across parties at the district level in order to achieve "notional" tied or reversed elections for the purpose of estimating partisan bias. We have pointed out that in Britain, the assumption of uniform swing is more appropriate for some parties than others.

Concluding Remarks

The agreement between the theoretical model and the empirical model is not only a validation of our theoretical strategy but also, more interestingly, a tool to explore how the mapping of votes to seats responds to changes in the territorial distribution of votes. Knowing the territorial distribution of party votes, our model predicts with a high degree of accuracy the expected majoritarian and partisan biases that derive from formal rules. In cases in which seat data are not available, (e.g., limited-franchise elections in 19th-century Europe), our model provides clear predictions. In cases where the territorial distribution of votes is likely to change, our model will anticipate the type of biases that can be expected.

While different electoral rules will yield different values for our rescaling parameters α_1 and α_2, the territorial distribution of votes should induce the same type of majoritarian and partisan biases in most electoral systems. That is, while we expect different natural majoritarian rates from different types of electoral rules, our model will fit all electoral rules where $\rho \geq 1$.

Our example using British data also demonstrated the usefulness of the model for teasing out subtleties of the relationship between the territorial vote distribution and the transformation of votes to seats in the real world. Because of its geography, Labour tends to suffer fewer seat losses during its periods in the wilderness, and significant seat gains during its periods of triumph. The Conservatives benefit from the partisan split on the left and enjoy the rather consistent seat premium associated with being a large and geographically dispersed party. As they have transformed themselves from a regionally concentrated to a dispersed party, the Liberals have endured large seat penalties. Thus, it is not surprising that in recent debates, most Labour and virtually all Conservative MPs favored the retention of the existing electoral system while Liberal Democrats have campaigned so vigorously against it.

References

Ansolabehere, Stephen, and James M. Snyder Jr. 2008. *The End of Inequality: One Person, One Vote and the Transformation of American Politics.* New York: Norton.

Borisyuk, Galina, Ron Johnston, Michael Thrasher, and Colin Rallings. 2010. "A Method for Measuring and Decomposing Electoral Bias for the Three-Party Case, Illustrated by the British Case." *Electoral Studies* 29: 733–45.

Brookes, R. H. 1959. "Electoral Distortion in New Zealand." *Australian Journal of Politics and History* 5(2): 218–33.

Brookes, R. H. 1960. "The Analysis of Distorted Representation in Two-Party, Single-Member Systems." *Political Science* 12(2): 158–67.

Calvo, Ernesto. 2009. "The Competitive Road to Proportional Representation: Partisan Biases and Electoral Regime Change under Increasing Party Competition." *World Politics* 61(2): 254–95.

Caramani, Danielle. 2000. *Elections in Western Europe since 1815: Electoral Results by Constituencies.* New York: Palgrave.

Chen, Jowei, and Jonathan Rodden. 2013. "Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures." *Quarterly Journal of Political Science* 8: 1–31.

Erikson, Robert. 2002. "Sources of Partisan Bias in U.S. Congressional Elections." *Political Geography* 21: 49–54.

Gelman, Andrew, and Gary King. 1990. "Estimating the Electoral Consequences of Legislative Redistricting." *Journal of the American Statistical Association* 85: 274–82.

Grofman, Bernard. 1983. "Measures of Bias and Proportionality in Seats-Votes Relationships." *Political Methodology* 9: 295–327.

Grofman, Bernard, William Koetzle, and Thomas Brunell. 1997. "An Integrated Perspective on the Three Potential Sources of Partisan Bias: Malappportionment, Turnout Differences, and the Geographic Distribution of Party Vote Shares." *Electoral Studies* 15(4): 457–70.

Gudgin, Graham, and Peter J. Taylor. 1979. *Seats, Votes and the Spatial Organisation of Elections.* London: Pion.

Johnston, Ronald J. 1979. *Political, Electoral and Spatial Systems.* Oxford: Oxford University Press.

Johnston, Ronald J. 2002. "Manipulating Maps and Winning Elections: Measuring the Impact of Malapportionment and Gerrymandering." *Political Geography* 21(1): 1–31.

Johnston, Ronald, Galina Borisyuk, Michael Thrasher, and Colin Rallings. 2012. "Unequal and Unequally Distributed Votes: Sources of Electoral Bias at Recent British General Elections." *Political Studies* 60: 877–96.

Johnston, Ronald J., DavidRossiter, and Charles Pattie. 1999. "Integrating and Decomposing the Sources of Partisan Bias: Brookes' Method and the Impact of Redistricting in Great Britain." *Electoral Studies* 18(3): 367–78.

Jones, Mark P., and Scott Mainwaring. 2003. "The Nationalization of Politics: An Empirical Measure and an Application to the Americas." *Party Politics* 9(2): 139–66.

Kendall, M. G., and A. Stuart. 1950. "The Law of the Cubic Proportion in Election Results." *British Journal of Sociology* 1(3): 183–96.

King, Gary. 1990. "Electoral Responsiveness and Partisan Bias in Multiparty Democracies." *Legislative Studies Quarterly* 15(2): 159–81.

King, Gary, and Robert X. Browning. 1987. "Democratic Representation and Partisan Bias in Congressional Elections." *American Political Science Review* 81(4): 1251–73.

Linzer, Drew A. 2012. "The Relationship between Seats and Votes in Multiparty Systems." *Political Analysis* 20: 400–416.

Rodden, Jonathan. 2013. The Long Shadow of the Industrial Revolution: Political Geography and the Representation of the Left. Unpublished manuscript, Stanford University.

Taagepera, Rein. 1973. "Seats and Votes: A Generalization of the Cube Law of Elections." *Social Science Research* 2: 257–75.

Taagepera, Rein. 1986. "Reformulating the Cube Law for Proportional Representation Elections." *American Political Science Review* 80(2): 489–504.

Taagepera, Rein. 1989. "Empirical Threshold of Representation." *Electoral Studies* 8(2): 105–16.

Taagepera, Rein. 2002. "Nationwide Threshold of Representation." *Electoral Studies* 21: 383–401.

Taagepera, Rein, and Matthew Soberg Shugart. 1993. "Predicting the Number of Parties: A Quantitative Model of Duverger's Mechanical Effect." *American Political Science Review* 87(2): 455–64.

Tufte, Edward. 1973. "The Relationship between Seats and Votes in Two-Party Systems." *American Political Science Review* 67: 540–54.

Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Table S1: Mean District Vote, Standard Deviation, and Coefficient of Variation (CV) in UK Elections
Table S2: Empirical Coefficient of Variation and Theoretical Coefficient of Variation in UK Elections
Figure S1: Majoritarian and Partisan Bias, UK Elections, Selected Parties, 1950–2010
Figure S2: Majoritarian and Partisan Bias, UK Elections, All Parties, 1950–2010
Appendix A: GINI Coefficients and the Coefficient of Variation
Appendix B: Descriptive Vote and Seat Data
Appendix C: Model Diagnostics and Assessments of Fit
Appendix D: Annotated WinBUGS Code